Expansion of the Newton potential and short–time dynamics of the Solar system

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Abstract: We analyze the motion of the eight principal planets around the Sun under Newton's inverse–square law of gravitation. In heliocentric coordinates, the orbit of each planet depends on six orbital elements, satisfying the Lagrangian Planetary Equations (LPE), i.e. the system of 48 first order ordinary differential equations.

In 1874 Simon Newcomb proved that any orbital element can be written as

$$(1) \qquad \alpha_0 t + \beta_0 + \sum A_{\mathbf{j}} \begin{matrix}\cos\\ \sin\end{matrix} \big(j_1(\alpha_1 t + \beta_1) + \cdots + j_{24}(\alpha_{24} t + \beta_{24})\big) \ ,$$

where t is the time, $\mathbf{j} = (j_1, \ldots, j_{24}) \in \mathbb{Z}^{24} - \{\mathbf{0}\}$ and $A_{\mathbf{j}}, \alpha_k, \beta_k$ are real constants. These expressions satisfy the LPE only formally.

On the other hand, the actual site of NASA suggests to reduce the above formulae to $\alpha_0 t + \beta_0 +$ (one additional term); this would be sufficient for practical observations at least for the next 1000 years.

We give an inter-medial explicit variant with 48 expression of type (1) but the multi-index $\mathbf{j}$ varies within a finite set. For any pair of planets we first expand the corresponding Newton's potential in double Fourier series in eccentric anomalies, and then convert the eccentric- to mean anomalies or mean longitudes. Finally we extract the most influential harmonics to obtain more accurate and long-time valid ephemerides.